Suggested solution of HW2

1. Clearly $\max A = \sup A = 1$ and $\min A$ does not exists. Suppose not, $a = \min A \in (0, 1]$. Then there is $a/2 \in (0, 1]$ which $a/2 < a$ which contradicts with minimality.

   Since for all $a \in (0, 1]$,
$$0 \leq a.$$
   And for any $\epsilon > 0$, $\epsilon/2 \in A$ and $\epsilon/2 < \epsilon$. Therefore 0 us the least lower bound of A.

3. For all $z \in X$,
$$f(z) + g(z) \leq \sup f(x) + \sup g(x).$$
   Taking sup on the right hand side give the conclusion. Strict inequality happen for example when $f(x) = \sin x$, $g(x) = \cos x$ and $X = \mathbb{R}$.

6. If $x \geq 0$, then $-1 \in A$ clearly. If $x < 0$, by Archimedean property, there is $N \in \mathbb{N}$ such that $N > -xn$. Therefore $A \neq \emptyset$. Clearly, A is bounded from above. By completeness of real number, there is a $\sup A = \bar{\kappa} \in \mathbb{R}$. We have $\bar{\kappa} \in \mathbb{Z}$ otherwise $[\bar{\kappa}] < \bar{\kappa}$ is a upper bound of A which is impossible. And we have
$$nx - 1 \leq \bar{\kappa} \leq nx \leq ny - 1.$$
   Therefore, $\bar{\kappa} + 1/n$ is a rational number in between x and y.